Algebra 1/Data Analysis

UNIT 7 Exponents and Exponential Functions	DAY 4
LESSON Exponential Growth Functions	
HOLT SECTION(S) 8.5-8.6	**McDOUGAL-LITTELL SECTION(S):** 8.5

OBJECTIVE(S)

- The student will recognize and graph exponential growth and decay functions.
- The student will interpret real-world situations using exponential functions.

CORE LEARNING GOAL(S)

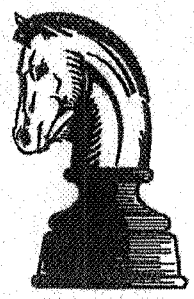
1.2.4 The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

TEACHER NOTE(S)

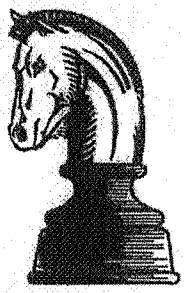
- This lesson is linked to Unit 3 Day 1 when nonlinear functions were discussed.
- Emphasize the equation and shape of the graph for this function.

SUGGESTED ACTIVITIES

- "Exponential Growth" (Student Resource Sheet 7.2): Have students create a table of the rice amounts on the first few squares. Students can enter data into lists and examine scatter plot. Discuss the graph and have students try different regression equations for model of best fit.
- "ECR 3" (Teacher Resource 7.4)
- "Space Debris Problem" (Student Resource Sheet 7.3)
- Core Plus Unit 6: Lesson 1 Investigations



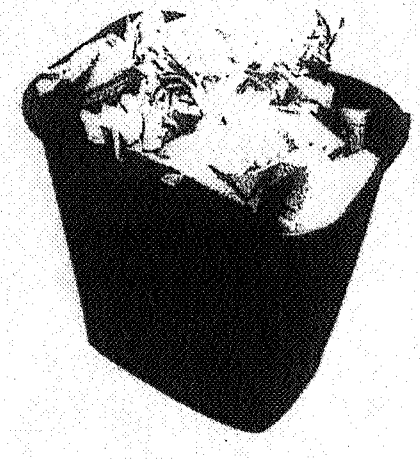
Exponential Growth



A wealthy king was rescued from danger by a quick thinking and brave soldier. The king wanted to honor the poor soldier, so he offered a very generous reward: a beautiful chessboard made of ivory and ebony and a set of gold chess pieces.

While the chess set was beautiful and valuable, the young man asked for a different reward. To help the poor people in his country, he asked the king to distribute rice from his storehouse—two grains for the first square of the chessboard, four grains for the second square, eight grains for the third square, sixteen grains for the fourth square, and so on. The king was pleased that he could keep his beautiful chessboard and repay the brave soldier with such a simple grant of rice to the poor. But he soon discovered that the request was not as simple as he thought.

Space Debris Problem



In 1993, scientists used radar to record a total of 7,000 debris objects in the Earth's orbit. Through the following years, they were able to determine that the amount of space debris is growing at a rate of 3% per year.

Let t = year, where t = 0 represents 1993 and let d = the debris total in the Earth's orbit.

1. Create a table recording the projected amount of space debris in the Earth's orbit between 1993 and 2005.

Year	Debris
0	7000
1	
2	

2. Create a scatter plot of the data on graph paper. Choose an appropriate scale and label axes.

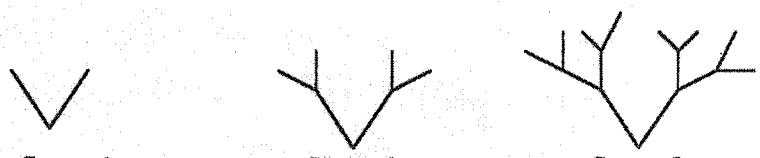
3. Write an equation to represent the amount of space debris in terms of the year.

4. Estimate the total projected amount of space debris in the year 2015.

5. Write a news brief to your local newspaper containing the information you were given about the space debris and your prediction. Include evidence to support your conclusion.

ECR 3

Fractals are patterns that are used in science to understand various forms in nature, including the branching structures of trees. The pattern starts with two branching line segments (Stage 1). Two branches are then added to the end of each previous branch.



Stage 1 Stage 2 Stage 3

- **In the space below, draw the branch structure for Stage 4.**

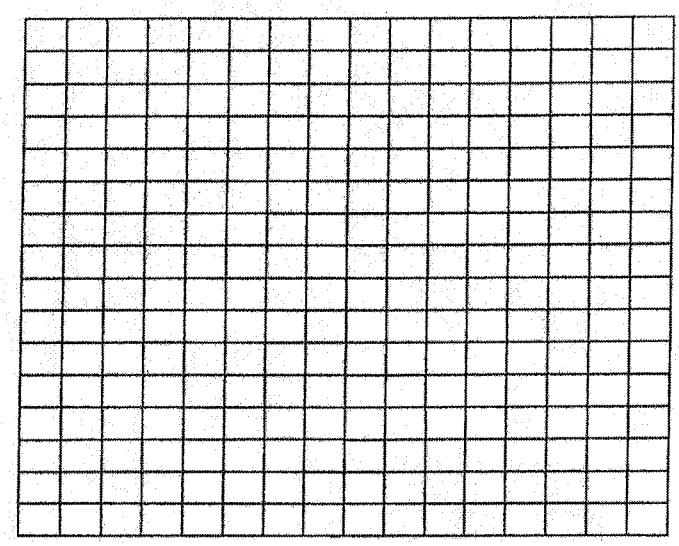
Stage 4

- **Complete the table below to show the number of new branches added at each of the first 6 stages.**

FRACTAL PATTERN

Stage Number	Number of New Branches
1	2
2	4
3	8
4	
5	
6	

- **Graph the number of new branches as a function of the stage number on the grid.**



- **Should the points on your graph be connected? Justify your answer.**

An Exponential Investigation

Mathematics is the study of patterns. In this investigation you will explore and compare several slightly different patterns. You will conduct the experiment that follows, record your data, analyze your data, and describe the pattern using a mathematical function, if possible. As the week develops, you will understand how this important pattern relates to our study of radioactive decay.

Before you begin this experiment make sure that you identify who within your group will do each of the jobs. Fill the name in beside the job description:

_____ **Tosser** – This person is responsible for tossing the objects described in the experiment.

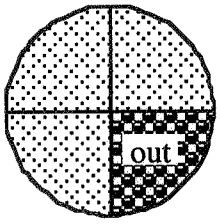
_____ **Remover** – This person is responsible for removing the objects described in the experiment.

_____ **Counter** -- This person is responsible for counting the remaining objects and recording this number in the data chart.

_____ **Reader** -- This person is responsible for reading the lab to the group and making sure the group works through the lab in a timely fashion.

Procedure:

1. The tosser should collect the assigned objects from the teacher – either:
   - thumb tacks
   - pennies
   - unpopped kernels of corn on a paper plate with one quarter shaded



2. The counter should count and record the number of objects and then place them in a cup.
3. The tosser should then shake the cup, and toss the objects onto a flat surface, or in the case of the corn, as evenly as possible into the plate.
4. The remover should remove the appropriate objects, either:
   - thumb tacks that have landed with the point sticking straight up
   - pennies that come up heads
   - kernels of corn that land in the shaded section of the plate.
5. The counter should count the number of objects that remain.
6. The counter should record the number of remaining objects in the data chart.
7. The tosser should gather the remaining objects, place them in the cup, and repeat steps 2-5 until there are fewer than 5 objects left to toss or until this has been repeated 14 times. It is important that you not record zero as the number of objects remaining.

Data Collection

Number of toss	Number of Objects Remaining
0	200
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

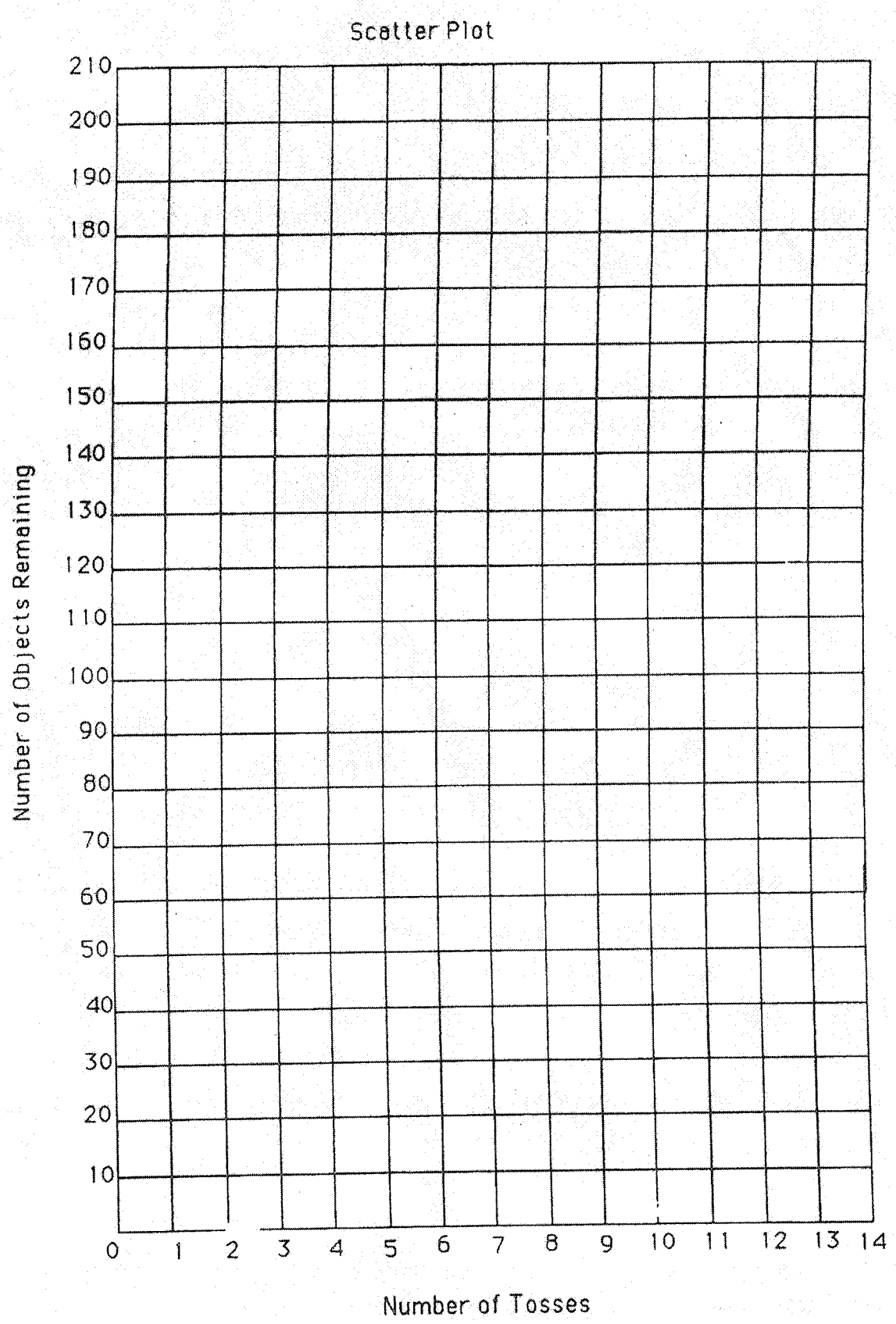
Clear L1 and L2 and turn off all plots before your begin.

1. Enter the data values in the TI-83 using L1 and L2.
2. Make a scatter plot.
3. Is this a function? If so, what type?
4. Use the TI-83 to determine a mathematical model. Give the equation. Now graph this equation using the TI-83.

   a) Select **STAT**, then **CALC**, then **0:ExpReg**.

   b) **ExpReg** will appear on your screen. You then type **L1, L2, Y1**. **Y1** is found under **VARS, Y-VARS, FUNCTION**

   c) Select **GRAPH** to display the graph of your exponential equation. Remember that this graph gives you the number of objects remaining (y), after a given number of tosses (x).


Scatter Plot
Number of Objects Remaining
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Number of Tosses

Algebra 1/Data Analysis

UNIT 7 Exponents and Exponential Functions	DAY 5
LESSON Exponential Decay Functions	
HOLT SECTION(S) 8.5-8.6	**McDOUGAL-LITTELL SECTION(S):** 8.6
OBJECTIVE(S) • The student will recognize and graph exponential growth and decay functions. • The student will interpret real-world situations using exponential functions.	**CORE LEARNING GOAL(S)** 1.2.4 The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

TEACHER NOTE(S)

- Discuss the differences between exponential growth and decay.
- This lesson may take 1-2 days depending on the time spent on activities.

- The requirements for the **High School Assessment** will have been met by the end of this lesson.

SUGGESTED ACTIVITIES

- McDougal Littell Activity 8.6: Developing Concepts: Investigating Exponential Decay p. 483
- "Paper Folding Activity" (Student Resource Sheet 7.4)
- "Exponential Decay Lab" (Student Resource Sheet 7.5)
- "An Exponential Investigation" (Student Resource Sheet 7.6)
- PBS Mathline HSMP-Rhinos and M&M's Lesson
- Core Plus Unit 6: Lesson 2 Investigations

Paper Folding

Fold a piece of paper in half and record the number of sections after each fold.

Number of Folds	Number of sections
0	
1	
2	
3	
4	
5	
6	

Make a scatterplot of your data. Write an equation for your data.

Exponential Growth:

Assume the full piece of paper has an area of 1. Fold the paper in half and determine the area of the smallest section after you've made the fold. Record the area.

Number of Folds	Area
0	
1	
2	
3	
4	
5	
6	

Make a scatterplot of your data. Write an equation for your data.

Exponential Decay:

Algebra 1/Data Analysis
Exponential Decay Lab

Purpose: Create, solve and graph situations involving exponential decay.

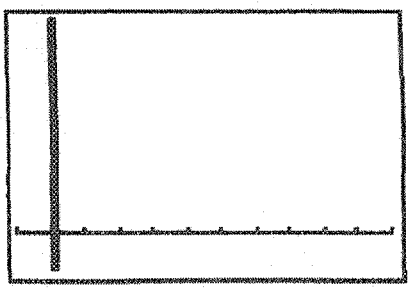
Materials: Dixie cup
Paper plate
M&Ms
TI-83 Calculator

Procedure:
1. Count and record the number of M&Ms in the dixie cup.
2. Dump the M&Ms on the paper plate. The ones that land "m" side up are the survivors.
3. Count and record the survivors and place them back in the dixie cup. Discard the other M&Ms.
4. Repeat steps 2 and 3 until the table is complete.

Data:

Dump Number	Number of survivors
0	
1	
2	
3	
4	
5	
6	
7	
8	

Graph:
1. Enter the data into L_1 and L_2 of your calculator.
2. Set **StatPlot 1** for a scatterplot. Press **Zoom 9** to graph. Sketch graph below. Describe the graph.



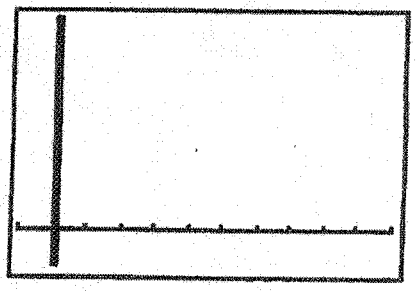
Equation for exponential decay:

1. Write an equation for your graph based on this:

$$y = \text{original number of M\&Ms} \cdot .5^x$$

$y =$ _____ $\cdot .5^x$

2. Enter the equation into Y= on your calculator and press graph. Sketch below. How well does it fit your data?

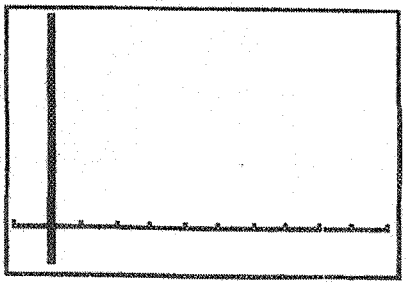


3. Replace .5 in the equation with other numbers and graph. Record each below and state if it creates a better fit or not.

Number tried	Better fit than .5?

4. Record the equation that had the best fit to your data, enter it into Y= and sketch the graph.

$y =$ _____ $\cdot$ _____^{x}



Conclusion:

Your equation and graph are examples of **exponential decay**. How are the graph and equation similar to exponential growth? How are they different?

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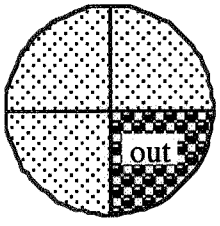

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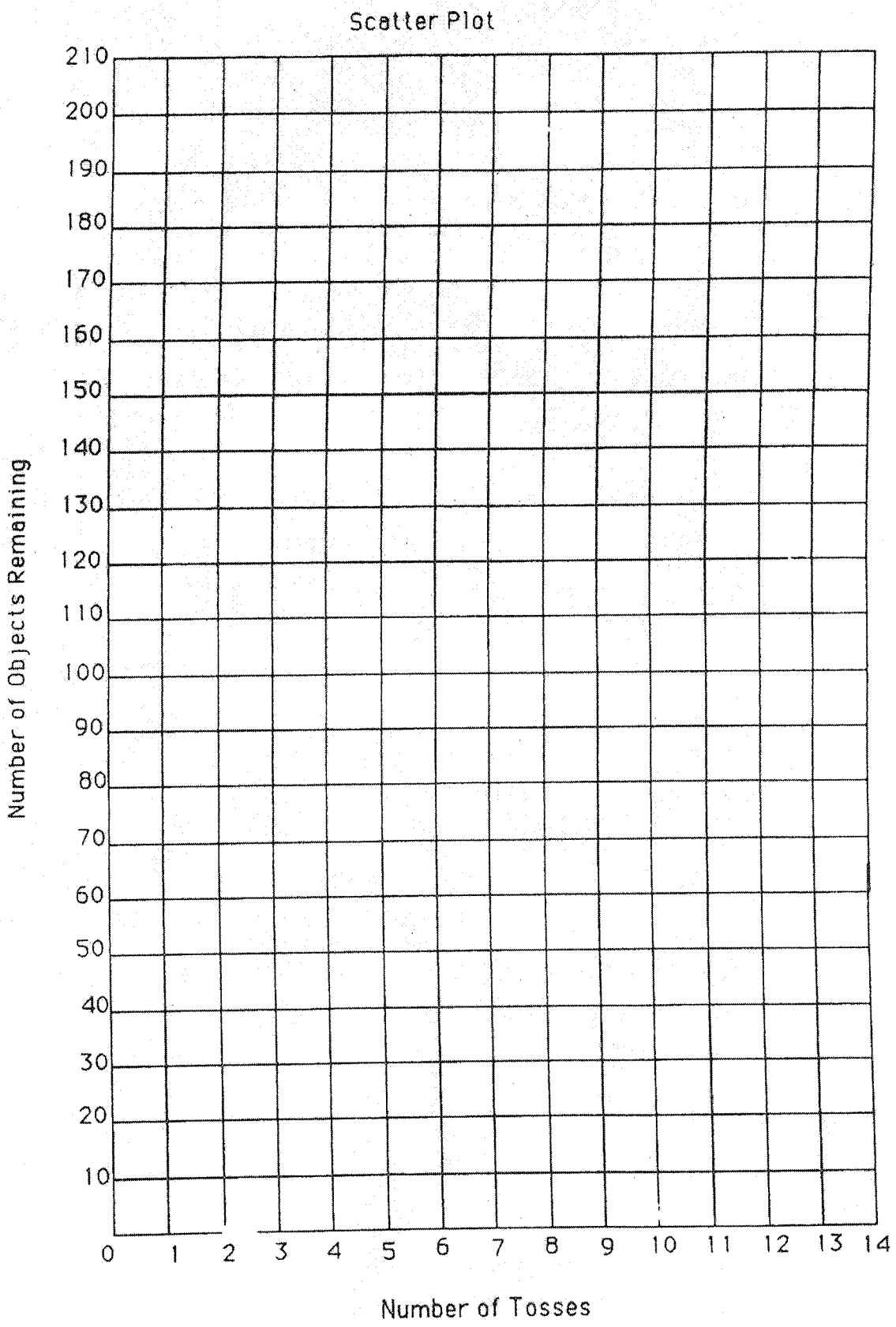
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Number of Tosses